

Will protons become gray at 13 TeV and 100 TeV?

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Abstract

It is shown that the regime of pp-interactions at 7 TeV is a critical one. The LHC data about elastic pp-scattering at 7 and 8 TeV are used to get some information about both elastic and inelastic profiles of pp-collisions. They are discussed in the context of two phenomenological models which intend to describe the high energy pp-data with high accuracy. Some predictions following from these models for LHC energy 13 TeV and for energy 95 TeV of the newly proposed collider are discussed. It is claimed that the center of the inelastic interaction region will become less dark with increase of energy albeit very slowly.

1 Introduction

The data of TOTEM collaboration [1, 2] about elastic scattering of protons at energies 7 and 8 TeV revived interest to its characteristics (for recent reviews see [3, 4]). They are important not only by themselves but also for getting some information about inelastic processes. One of the most exciting findings from the data is the completely dark profile of central inelastic collisions [5, 6]. The knowledge about it has been obtained directly from the unitarity condition. It states that the total probability of all outcomes of proton collisions must be equal 1 at a given energy and relates the elastic and inelastic amplitudes. Inserting there the experimental data about elastic scattering one gets some conclusions about inelastic events. The saturation limit of the inelastic profile equal 1 implies the complete darkness of the interaction region.

2 Profiles

By the profile of collisions we mean the impact parameter b distribution of the strength of the interaction. The impact parameter is defined as the

shortest transverse distance between the trajectories of the protons' centers. In this representation the unitarity condition looks as

$$G(s, b) = 2\Gamma(s, b) - \Gamma^2(s, b). \quad (1)$$

$\Gamma(s, b)$ is defined as the Fourier - Bessel transform of the elastic scattering amplitude $f(s, t)$ which depends on energy s and transferred momentum t . It retranslates the transferred momentum data to the transverse impact parameter space features and is written as

$$\Gamma(s, b) \approx \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| \text{Im} f(s, t) J_0(b\sqrt{|t|}). \quad (2)$$

The real part of the amplitude is small and will be neglected so that $\text{Im} f(s, t) \approx \sqrt{d\sigma/dt}$.

The left-hand side of Eq. (1) describes the impact parameter profile of inelastic collisions of protons. It satisfies the inequalities $0 \leq G(s, b) \leq 1$ and determines how absorptive is the inelastic interaction region depending on the impact parameter (with $G = 1$ for the full absorption and $G = 0$ for the complete transparency). The profile of elastic processes is determined by the subtrahend in Eq. (1). If $G(s, b)$ is integrated over the impact parameter, it leads to the cross section of inelastic processes. The terms on the right-hand side would produce the total cross section and the elastic cross section.

The elastic differential cross section is especially large at small transferred momenta within the so-called diffraction cone where it decreases exponentially with $|t|$. Thus, the diffraction cone contributes mostly to the Fourier - Bessel transform of the amplitude. Using the above formulae, one can write the dimensionless Γ as

$$\Gamma(s, b) = \frac{\sigma_{tot}}{8\pi} \int_0^\infty d|t| \exp(-B|t|/2) J_0(b\sqrt{|t|}). \quad (3)$$

Here, the diffraction cone approximation is inserted:

$$\frac{d\sigma}{dt} = |f(s, t)|^2 \approx \frac{\sigma_{tot}^2}{16\pi} \exp(-B|t|). \quad (4)$$

Herefrom, one calculates

$$\Gamma(s, b) = \zeta \exp\left(-\frac{b^2}{2B}\right), \quad (5)$$

where we introduce the dimensionless ratio of the cone slope (or the elastic cross section) to the total cross section

$$\zeta = \frac{\sigma_{tot}}{4\pi B} \approx \frac{4\sigma_{el}}{\sigma_{tot}}. \quad (6)$$

Possible small deviations from the exponential behavior inside the cone do not practically change the value of the integral contribution. The differential cross section is quite small outside the diffraction peak and does not influence the impact parameter profile G .

Thus, we get

$$G(s, b) = \zeta \exp\left(-\frac{b^2}{2B}\right) \left(2 - \zeta \exp\left(-\frac{b^2}{2B}\right)\right). \quad (7)$$

The inelastic profile $G(s, b)$ is directly obtained from the unitarity condition as a difference of two Gaussian exponentials. The exponents differ by a factor of two. The shapes of the profiles are completely determined by the slope $B(s)$ and the total cross section $\sigma_{tot}(s)$. They have been computed at different energies using the corresponding experimental data. From ISR (62.5 GeV) to LHC (7 TeV) energies the inelastic interaction region of protons becomes darker and extends to larger impact parameters. That supports earlier expectations that the protons become more black, edgier and larger (BEL). At the LHC energy 7 TeV the inelastic profile is extremely dark up to about 0.5 fm and its width is about 1.5 fm while at the ISR energy 62.5 GeV the darkness was slightly above 0.9 at the center and decreased twice already at about 0.7 fm.

The elastic profile given by the subtrahend in (7) is comparatively more narrow. The inelastic processes are always more peripheral than the elastic ones. The ratio of the average values of the squared impact parameters is given by

$$\frac{\langle b_{inel}^2 \rangle}{\langle b_{el}^2 \rangle} = \frac{8 - \zeta}{4 - \zeta} \quad (8)$$

as is easily seen from their profiles normalized by the corresponding cross sections.

For central collisions with $b = 0$ one gets

$$G(s, b = 0) = \zeta(2 - \zeta). \quad (9)$$

This formula is very important. Herefrom, it follows that the darkness at the very center $b = 0$ is fully determined by the single parameter ζ , i.e. by the

ratio of experimentally measured characteristics - the width of the diffraction cone B (or σ_{el}) to the total cross section. Its energy evolution defines the evolution of the absorption value.

As a function of energy, the value of ζ changes in the range of ISR to LHC from 0.66 to 1.0 (with intermediate values of 0.8 at $S\bar{p}pS$ at 546 GeV and 0.9 at Tevatron at 1.8 TeV if the proton-antiproton data are included). Therefore, compared to ISR data, the darkness of central collisions strongly increases at the LHC. The inelastic interaction region becomes completely black at impact parameters b up to 0.5 fm [7]. The interaction region is completely absorptive ($G(s, 0) = 1$) in the center only at $\zeta = 1$ and the absorption diminishes for other values of ζ . Eq. (9) shows that the darkness at the center changes extremely slowly for values of ζ close to 1. The decline by $\pm\epsilon$ from 1 in ζ leads to corrections of the order of ϵ^2 in $G(s, b = 0) = 1 - \epsilon^2$. Therefore, the regime of pp-interactions at 7 TeV can be considered as a critical one.

That has some important consequences for inelastic processes indicating that high density gluon configurations in the interaction region play a crucial role for high multiplicity events at LHC energies [8]. For example, it was noticed at the CMS studies that experimental results at 7 TeV on jet production in rare events with the very high charged particle multiplicities above 70 differ from the predictions of the widely used Monte Carlo models PYTHIA and HERWIG. The production of jets is a typical feature of such events. The jets are mostly produced at central collisions where the density of partons (or strings) is high so that multiparton interactions become important and the interactions themselves are strong enough. It has been shown that at the very high multiplicities the rate of jet production is not described by the geometry of pp-interactions. These processes become dominated by rare central collisions, where some special high-density gluon fluctuations appear inside colliding protons. Therefore it looks likely that one should include more dense gluon states in Monte Carlo models to get an agreement with experiment.

3 Expectations

What can we expect at higher energies?

The only guesses can be obtained from the extrapolation of results at lower energies to new regimes even though our experience shows how indefi-

nite and even erroneous they can be as it often happened.

First, one may assume that ζ will increase without crossing 1 but approaching it asymptotically. That would imply that its precise value at 7 TeV is still slightly lower than 1 within the present experimental errors. Then the inelastic profile will be quite stable with slow approach to the complete blackness at central collisions and steady increase of its range. That is a kind of "the black stick" if one implies rather long longitudinal distances as it is commonly believed for the picture with wee partons.

Another, more intriguing possibility, discussed below in the framework of some phenomenological models, is the further increase of ζ above 1. Then the darkness at the very central collisions $G(s, b = 0)$ diminishes in accordance with Eq. (9). The center becomes more transparent. The dip should appear inside the plateau with a minimum value at $b = 0$. The black bump at the center observed at 7 TeV transforms to the toroid-like structure with somewhat lower darkness at the center and maximum blackness equal 1 at some peripheral impact parameter b_m . This dependence is very slow near $\zeta = 1$ so that the darkness at the center would only become smaller by 6% if ζ increases to 1.2. Therefore one can hardly expect the immediate drastic changes with increase of LHC energies. Nevertheless, the forthcoming TOTEM+CMS results on elastic scattering at 13 TeV can be very conclusive about the general trend if the precise values of the diffraction cone slope B and the total cross section σ_{tot} become available and the corresponding value of ζ happens to exceed 1.

This tendency is supported by recent phenomenological models [9, 10]. In one of them (KFK), the detailed form of the elastic scattering amplitude is prescribed. The diffraction cone region at small $|t|$ and further Orear-type regime at larger $|t|$ with exponential decrease in $\sqrt{|t|}$ are parameterized and successfully compared with present experimental data. By passing, we note that the real part of the amplitude computed in the model is extremely small within the diffraction cone and even passes zero inside it. That confirms our assumption about it. In another model (FMS), the energy behaviour of the ratio of the elastic and total cross sections is studied. According to Eq. (6), the energy behaviour of ζ has been directly modeled. Both models intend to describe quite well the presently available experimental data and give some predictions for higher energies.

In the Table, the values of the slope $B(s)$ and the total cross section $\sigma_{tot}(s)$ available in KFK are reproduced. We have calculated the corresponding values of ζ and $G(s, b = 0)$ for KFK extending the range of energies up to

Table. The energy behavior of $\zeta(s)$ and $G(s, b = 0)$.

	\sqrt{s}, TeV	$\zeta(s)$	$G(s, 0)$	$b_m(s), \text{fm}$	$B(s), \text{GeV}^{-2}$	$\sigma_{tot}(s), \text{mb}$
<i>TOTEM</i> [1] [2]	7	1.0128	0.9998	0.14	19.9	98.6
	8	1.0189	0.9996	0.17	20.4	101.7
<i>KFK</i> [9]	7	1.0133	0.9998	0.14	19.9	98.65
	8	1.0215	0.9995	0.18	20.21	101.00
	13	1.0524	0.9973	0.29	21.35	109.93
<i>computed</i>	95	1.1497	0.9776	0.53	27.10	152.43
	1000	1.2393	0.9428	0.74	35.50	215.24
	∞	1.4399	0.8060	∞	∞	∞
<i>FMS</i> [10]	8	1.0300	0.9991			
	13	1.0554	0.9969			
	95	1.1231	0.9848			

its asymptotics. The values of ζ and $G(s, b = 0)$ have also been computed for FMS. It is clearly seen that both models favor slight decrease of the inelastic profile at the center with the shift of maximum blackness equal 1 to peripheral impact parameters $b_m(s)$ which are shown in a separate column.

Both models predict slightly faster increase of ζ with energy from 7 to 8 TeV if compared to the TOTEM data albeit within experimental errors. The values of $G(s, 0)$ do not change, in practice, even though slight tendency to decrease in the fourth (!) digit is shown in the Table. At the energy 13 TeV the value of ζ differs from that at 7 TeV only less than by 4%. Correspondingly, the darkness is different at the third digit. That puts some hard problems for experimenting at 13 TeV. Nevertheless, new experimental data at 13 TeV would show the general qualitative trend if their accuracy is high enough.

What concerns higher energies, the above tendency persists. The central region becomes more gray. It may be noticed in the data from the newly proposed 100 TeV-collider as shown in the Table in rows at 95 TeV if one trusts the models' predictions. Due to the assumptions of some saturation in asymptotics, used in the models, the predictions evolve extremely slowly with energy so that even the asymptotical value of $G(s, 0)$ is close to 0.8 as compared to the absolute blackness 1 at peripheral impact parameters $b_m(s)$ also shown in the Table.

It is hard to foresee any special features of inelastic processes if the evo-

lution will be that slow. The (almost) black plateau will become somewhat enlarged in size. Therefore the jet cross sections due to central collisions will increase as well. The inelastic profile will become even more peripheral and the role of peripheral collision will increase.

In principle, this tendency will be preserved until the value of ζ becomes equal to 2 which erroneously is often called as the black disk regime. Then at the corresponding energy s_0 the center of the inelastic interaction region becomes completely transparent $G(s_0, b = 0) = 0$. It means that elastic processes will only dominate at the very central collisions, and the inelastic events will originate at more peripheral regions. Surely, this possibility looks quite opposite to our intuition. Nevertheless, it can happen according to the above dependence of the central darkness $G(s, b = 0)$ on ζ . No model predicts the fast rise of ζ to values close to 2. Their preferred asymptotic values of ζ are less than 1.5 as shown in the Table.

The values of ζ larger than 2 would correspond to another branch of the solution of the unitarity equation which describes the backward scattering (the reflective mode). Its realization in the near future looks very improbable even in the models and we do not consider it here.

To conclude, the latest findings of TOTEM and CMS contributed much to understanding of the shape of the interaction region of colliding protons. Its structure changes with increase of their energies and one can await that some new features will be noticed at higher energies.

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